COMBUSTION OF A LIQUID FUEL DROPLET IN AN ACOUSTIC FIELD

V. E. Nakoryakov

Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 5, pp. 650–656, 1967

UDC 536.46

The rate of combustion of a liquid fuel droplet in an acoustic field is investigated analytically for certain limiting values of the dimensionless numbers characterizing the process. An expression relating the burning rate and the acoustic parameters is derived.

The results of a theoretical and experimental investigation of heat and mass transfer processes in an acoustic field were presented in [1–3].

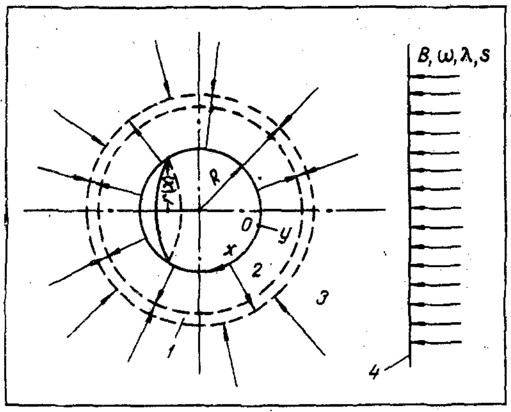

Fig. 1. Diagram of the process: 1) reaction zone; 2) fuel vapor diffusion zone; 3) oxygen diffusion zone; 4) acoustic wave front.

It was established that there is a very characteristic relation between the transfer coefficients and the parameters of the acoustic field. Thus, in the case of mass transfer from a sphere one such relation has the form

$$\mathrm{Nu} = 1.3\,\frac{B}{\sqrt{\omega D}}.$$

It has recently been shown experimentally that it is possible to create high-intensity combustion chambers using an acoustic field as a means of accelerating the combustion of the individual fuel droplets and particles [4].

This paper presents the results of a theoretical analysis of the process of combustion of an individual droplet of liquid fuel. The problem of the combustion of a droplet is more complicated than the problem of "pure" mass transfer between a medium and a sphere owing to the need to take into account the chemical reactions and because of the effect of the transverse flow of fuel vapor on the hydrodynamics of the process.

We will consider a droplet of evaporating fuel of radius R in a medium disturbed by a plane acoustic wave (Fig. 1). The acoustic wave is characterized by the parameters B, ω, $s = B/\omega$, λ (here, s is the amplitude of displacement of particles of the medium in the acoustic wave).

Fuel vapor diffuses from the surface of the droplet to the reaction zone, while oxygen diffuses to the reaction zone from the surrounding medium. The heat released in the reaction zone vaporizes the fuel and heats the surrounding masses of gas.

As usual, we assume that the mixture is binary, that the physical parameters of the medium do not depend on the concentration and temperature fields, that the temperature of the droplet surface is equal to the boiling point, and that the droplet preserves its spherical shape during combustion.

We will now consider the problem for certain limiting values of the dimensionless numbers that characterize the process. As in [3] we assume that $s/R \ll 1$, $\lambda/R \gg 1$, and $\omega R^2/\nu \gg 1$.

In the coordinate system Oxy (Fig. 1) the equations of the dynamic boundary layer are written as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}, \tag{1}$$

$$\frac{\partial (ur)}{\partial x} + \frac{\partial (vr)}{\partial y} = 0, \tag{2}$$

$$v = v_w \quad \text{at} \quad y = 0,$$
$$u = V = F(x)\cos\omega t \quad \text{at} \quad y = \infty$$

with the boundary conditions

$$u = 0 \quad \text{at} \quad y = 0.$$

Proceeding as in [3], we present the solution in the form of a sum of the stationary and fluctuating components: $u = u_0 + u'$, where u_0 is the stationary and u' the fluctuating component of velocity.

To determine u' we obtain the equation

$$\frac{\partial u'}{\partial t} + v_w\frac{\partial u'}{\partial y} - \nu\frac{\partial^2 u'}{\partial y^2} = \frac{\partial V}{\partial t} \tag{3}$$

with the boundary conditions

$$u = V = F(x)\cos\omega t \quad \text{at} \quad y = \infty,$$
$$u = 0 \quad \text{at} \quad y = 0.$$

The solution of (3) has the form

$$u' = F(x)\Bigg\{\cos\omega t - \exp\Bigg[y\left(\frac{v_w}{2\nu} - \right.$$

$$\left. - \sqrt[4]{\left(\frac{v_w}{\nu}\right)^4 + \left(\frac{\omega}{\nu}\right)^2}\cos\frac{\operatorname{arctg}\dfrac{4\nu\omega}{v_w^2}}{2}\right]\times$$

$$\times \cos\left[\omega t - y\sqrt[4]{\left(\frac{v_w}{\nu}\right)^4 + \left(\frac{\omega}{\nu}\right)^2} \times \times \sin\frac{\operatorname{arctg}\frac{4\nu\omega}{v_w^2}}{2}\right]\Bigg\}. \tag{4}$$

An analysis of solution (4) shows that when $v_w \ll \ll 2(\omega\nu)^{1/2}$ it goes over into the well-known relation

$$u' = F(x)\cos\left[\omega t - \exp(-y\sqrt{\omega/2\nu}) \times \times \cos\left(\omega t - y\sqrt{\frac{\omega}{2\nu}}\right)\right]. \tag{5}$$

Since the condition $v_w \ll 2(\omega\nu)^{1/2}$ is usually satisfied when a liquid droplet burns, in what follows we will employ only relation (5).

For the longitudinal component of the stationary velocity we obtain the equation

$$\nu\frac{\partial^2 u_0}{\partial y^2} = \left\langle u'\frac{\partial u'}{\partial x}\right\rangle + \left\langle v'\frac{\partial u'}{\partial y}\right\rangle - \left\langle V\frac{\partial V}{\partial x}\right\rangle, \tag{6}$$

where $\langle\,\rangle$ is the sign of averaging with respect to time.

The solution of Eq. (6) does not depend on the presence of "injection" and is given in [3].

The transverse velocity component is determined from the continuity equation for the stationary velocity component as

$$v_0 = v_w - \int\limits_0^y \frac{\partial(u_0 r)}{\partial x}\partial y. \tag{7}$$

Thus, the velocity field of the secondary flows in the vicinity of the sphere can be determined in the form of a linear superposition of two flows: the secondary flows without injection and injection itself.

A similar result for a somewhat different case was obtained by Kestin [5].

As in the case of zero transverse mass flow, the secondary flows converge on the equator of the sphere (Fig. 2), forming a very thin boundary layer of thickness $\delta_{\text{dyn}} = (2\nu/\omega)^{1/2}$ at the surface of the droplet.

At the outer edge of the dynamic boundary layer

$$u_0 = V_0 = -\frac{3}{4}\frac{F(x)}{\omega}\frac{\partial F(x)}{\partial x} - \frac{1}{2}\frac{F^2(x)}{\omega r(x)}\frac{\partial r(x)}{\partial x}.$$

For potential flow around a sphere

$$F(x) = \frac{3}{2}B\sin\frac{x}{R}, \quad r(x) = R\sin\frac{x}{R}.$$

In the coordinate system Oxy (Fig. 1)

$$V_0 = -A\sin\frac{2x}{R},$$

and in the coordinate system O'xy (Fig. 2)

$$V_0 = A\sin\frac{2x}{R}, \tag{8}$$

where

$$A = 1.4\frac{B^2}{\omega R}.$$

In the coordinate system O'xy (Fig. 2) the variable radius of the sphere is written as

$$r = R\cos\frac{x}{R}.$$

The equation describing the combined development of the process of diffusion, heat conduction, and heat

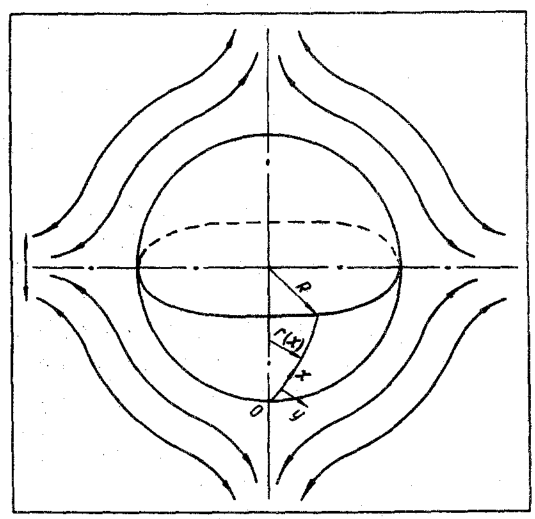

Fig. 2. Diagram of secondary flows in the vicinity of the droplet.

release in the gas volume adjacent to the droplet is written in the coordinate system O'xy (Fig. 2) as [6]

$$\frac{H}{\chi}\left[D\left(\frac{\partial^2 m_0}{\partial y^2}\right) - u\frac{\partial m_0}{\partial x} - v\frac{\partial m_0}{\partial y}\right] + \left[a\left(\frac{\partial^2 I}{\partial y^2}\right) - u\frac{\partial I}{\partial x} - v\frac{\partial I}{\partial y}\right] = 0. \tag{9}$$

Here,

$$I = c_p T.$$

When the Lewis-Semenov number $D/a = 1$, a Schwab-Zel'dovich transformation is possible and Eq. (9) can be written [6–8]

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D\frac{\partial^2 b}{\partial y^2}, \tag{10}$$

where b is the Schwab-Zel'dovich variable, equal to the total enthalpy of the mixture divided by the heat of vaporization of the fuel:

$$b = \frac{m_0 H\chi^{-1} + c_p T}{Q}.$$

Equations (9) and (10) have been written on the assumption that heat and mass transfer is realized by the secondary flows while the velocity fluctuations and fluctuations of the variable b have little effect on the average heat and mass transfer. In [1–3] it was shown that this assumption is rigorously satisfied in the "pure" process, in the absence of chemical reactions and a transverse mass flow. An analogous proof for the problem of droplet combustion has been omitted owing to the clumsiness of the calculations.

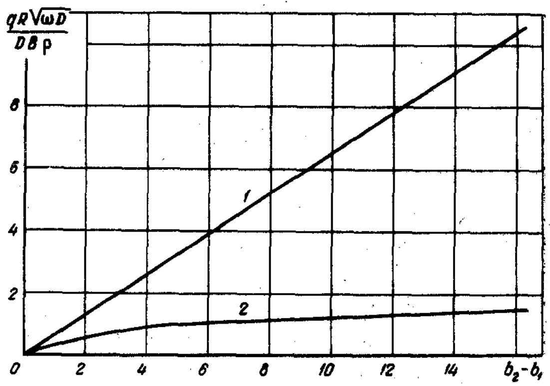

Fig. 3. Effect of transverse mass flow on the rate of droplet combustion: 1) calculated from Eq. (23); 2) (22).

We also assume that the thermal and diffusion resistance of the dynamic boundary layer is small and that the longitudinal component of velocity in the diffusion boundary layer is given by expression (8). The validity of this assumption was also confirmed in [1–3].

We now rewrite Eq. (10) in the Mises variables x, ψ. The stream function ψ is determined from the relations

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y}, \quad v = v_w - \frac{1}{r}\frac{\partial\psi}{\partial x}.$$

We obtain the equation

$$\frac{\partial b}{\partial x} = DV_0 r^2 \frac{\partial^2 b}{\partial\psi^2} - v_w r\frac{\partial b}{\partial\psi} \tag{11}$$

with the boundary conditions

$$\begin{aligned} b &= b_1 \quad \text{at} \quad \psi = 0, \\ b &= b_2 \quad \text{at} \quad \psi = \infty. \end{aligned} \tag{12}$$

Introducing the new variable $\theta = \int\limits_0^x V_0 r^2 dx$, we obtain the equation

$$\frac{\partial b}{\partial\theta} = D\frac{\partial^2 b}{\partial\psi^2} - \frac{v_w}{V_0 r}\frac{\partial b}{\partial\psi} \tag{13}$$

with the boundary conditions (12).

At very low evaporation rates (13) goes over into the equation

$$\frac{\partial b}{\partial\theta} = D\frac{\partial^2 b}{\partial\psi^2},$$

whose solution has the form

$$b = b_2 + (b_1 - b_2)\left[1 - \operatorname{erf}\frac{\psi}{2\sqrt{D\theta}}\right]. \tag{14}$$

Since [8]

$$v_w = D\left(\frac{\partial b}{\partial y}\right)_{y=0},$$

from (14) we have

$$v_w = 1{,}59(b_2 - b_1)\sqrt{\frac{AD}{R}} \times \cos^2\frac{x}{R} \Big/ \sqrt{1 + \cos^2\frac{x}{R}}; \tag{15}$$

It is natural to assume that in all cases

$$v_w = v_* \cos^2\frac{x}{R} \Big/ \sqrt{1 + \cos^2\frac{x}{R}}. \tag{16}$$

Substituting (16) in (13), we arrive at the equation

$$\frac{\partial b}{\partial\theta} = D\frac{\partial^2 b}{\partial\psi^2} - \frac{k}{\sqrt{\theta}}\frac{\partial b}{\partial\psi}, \tag{17}$$

where

$$k = \sqrt{v_*^2 R/8A}.$$

Introducing the new variables $b = b(\eta)$, $\eta = \psi/\theta^{1/2}$, we obtain the equation

$$D\left(\frac{\partial^2 b}{\partial\eta^2}\right) - \left(k - \frac{1}{2}\eta\right)\frac{\partial b}{\partial\eta} = 0 \tag{18}$$

with the boundary conditions

$$\begin{aligned} b &= b_1 \quad \text{at} \quad \eta = 0, \\ b &= b_2 \quad \text{at} \quad \eta = \infty. \end{aligned}$$

The solution of Eq. (18) has the form

$$b = b_1 + (b_1 - b_2)\left[\operatorname{erf}\left(\frac{\psi}{2\sqrt{D\theta}} - \frac{k}{\sqrt{D}}\right) + \operatorname{erf}\frac{k}{\sqrt{D}}\right] \Big/ \left(1 + \operatorname{erf}\frac{k}{\sqrt{D}}\right). \tag{19}$$

From (19) it is easy to obtain an expression for the dimensionless rate of transverse flow of fuel vapor:

$$\frac{v_w R}{D} = \sqrt{\frac{8AR}{\pi D}}(b_2 - b_1) \times \frac{\cos^2(x/R)}{\sqrt{1 + \cos^2(x/R)}}\,\frac{\exp(-v_*^2 R/8AD)}{1 + \operatorname{erf}\sqrt{v_*^2 R/8AD}}. \tag{20}$$

For determining the unknown v_* we have the equation

$$\beta = \frac{b_2 - b_1}{\sqrt{\pi}}\,\frac{\exp - \beta^2}{1 + \operatorname{erf}\beta}, \quad \beta = \sqrt{\frac{v_*^2 R}{8AD}}. \tag{21}$$

On average for the droplet we have

$$\frac{qR}{D\rho} = 1.17\frac{B}{\sqrt{\omega D}}\beta, \tag{22}$$

where q is the amount of fuel evaporating in unit time from unit surface, and ρ is the density of the fuel vapor.

At low "injection" rates Eq. (22) goes over into the relation obtained in [3]:

$$\frac{qR}{D\rho} = 0.65 \frac{B}{\sqrt{\omega D}} (b_2 - b_1). \qquad (23)$$

The results of calculations based on (22) and (23) are presented in Fig. 3. It is clear that a transverse flow of evaporating fuel has an important effect on the process of combustion of the individual fuel droplet in an acoustic field, reducing the rate of surface evaporation.

Neither chemical reactions in the droplets nor a transverse mass flow affect the nature of the relationship between the dimensionless transport numbers and the parameters of the acoustic field. As before, the dimensionless evaporation rate depends linearly on the amplitude of the particle velocity and is inversely proportional to the square root of the oscillation frequency.

NOTATION

B is the amplitude of particle velocity in acoustic wave; ω is the oscillation frequency; D is the diffusion coefficient; s is the displacement amplitude; λ is the wavelength of acoustic oscillations; u is the longitudinal velocity component; v is the transverse velocity component; V is the velocity at outer edge of boundary layer; x is the longitudinal coordinate; y is the transverse coordinate; v_W is the vapor velocity at droplet surface; r(x) is the variable droplet radius; ν is the kinematic viscosity; R is the droplet radius; A is the characteristic secondary flow velocity; m_0 is the dimensionless mass concentration; a is the thermal diffusivity; I is the enthalpy; ρ is the density; H is the calorific value; c_p is the specific heat; χ is the stoichiometric factor; b is the Schwab-Zel'dovich variable; Q is the heat of vaporization; ψ is the stream function; θ and η are the independent variables; k is the injection parameter; β is the perturbing factor characterizing effect of injection and chemical processes; q is the amount of evaporating fuel.

REFERENCES

1. A. P. Burdukov and V. E. Nakoryakov, PMTF [Journal of Applied Mechanics and Technical Physics], no. 1, 1965.
2. A. P. Burdukov, E. G. Zaulichnyi, and V. E. Nakoryakov, Izv. SO AN SSSR, ser. tekhnicheskikh nauk, no. 6, 2, 1965.
3. A. P. Burdukov and V. E. Nakoryakov, PMTF [Journal of Applied Mechanics and Technical Physics], no. 2, 1965.
4. O. G. Roginskii, Akusticheskii zhurnal, no. 2, 1961.
5. J. Kestin and L. Persen, Appl. Sci. Res., section A, **11**, no. 4–6, 1963.
6. D. B. Spalding, Fundamentals of Combustion Theory [Russian translation], Gosenergoizdat, Moscow, 1959.
7. H. Emmons, Voprosy raketnoi tekhniki, no. 6, 1956.
8. L. Lees, collection: Gasdynamics and Heat Transfer in the Presence of Chemical Reactions [Russian translation], IL, 1962.

14 July 1966

Institute of Thermophysics of the Siberian Division AS USSR, Novosibirsk